## Cheming cheatsheet

A handy guide containing the most important equations that a chemical engineer will need in practical life. For a few selected equations, their derivation process is outlined, but only in very brief points. Czech names are in violet.
Unit conversions are not included; see this tool that will tackle anything you come across. By Jiří Zbytovský in 2023

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## Reaction quantification

$\mathrm{E}, \mathrm{P}$ stands for an extensive quantity for educts (reaktanty) and products, such as $n_{i}, m_{i}, \dot{n}_{i}$. When certain conditions are met, also intensive quantities may be balanced: $c_{i}, p_{i}, w_{i}$ etc... $\nu_{i}$ is the stoichiometric coefficient of a component $i$.

Extent of reaction (reakční rozsah) for a component $i$

$$
\begin{equation*}
\xi=\frac{n_{i}-n_{i}^{0}}{\nu_{i}} \quad \xi_{V}=\frac{n_{i}-n_{i}^{0}}{V \nu_{i}} \tag{1}
\end{equation*}
$$

## Conversion

$$
\begin{equation*}
X=\frac{E^{0}-E}{E^{0}} \tag{2}
\end{equation*}
$$

Yield (výtěžek)

$$
\begin{equation*}
Y=\frac{P}{E^{0}} \frac{\nu_{\mathrm{E}}}{\nu_{\mathrm{P}}}=X \cdot S \tag{3}
\end{equation*}
$$

Selectivity (of one reaction with st. coeffs $\nu_{\mathrm{E}}, \nu_{\mathrm{P}}$ against all other reactions)

$$
\begin{equation*}
S=\frac{P}{E^{0}-E} \frac{\nu_{\mathrm{E}}}{\nu_{\mathrm{P}}} \tag{4}
\end{equation*}
$$

Relative selectivity (of two parallel reactions $\mathrm{E} \rightarrow \mathrm{P}, \mathrm{P} *$ with st. coeffs $\nu_{\mathrm{P}}, \nu_{\mathrm{P} *}$ )

$$
\begin{equation*}
S_{\mathrm{P} / \mathrm{P} *}=\frac{P}{P *} \frac{\nu_{\mathrm{P}} *}{\nu_{\mathrm{P}}}=\frac{r_{\mathrm{P}}}{r_{\mathrm{P} *}} \tag{5}
\end{equation*}
$$

## Reaction kinetics

Various definitions of reaction rate (reakční rychlost):

$$
\begin{align*}
R & =\frac{\mathrm{d} \xi}{\mathrm{~d} \tau}  \tag{6}\\
R_{i} & =R \nu_{i}  \tag{7}\\
r & =\frac{\mathrm{d} R}{\mathrm{~d} V}=\frac{\mathrm{d} \xi_{V}}{\mathrm{~d} \tau}=\frac{1}{V \nu_{i}} \frac{\mathrm{~d} n_{i}}{\mathrm{~d} \tau}  \tag{8}\\
r_{i} & =r \nu_{i}=\frac{\mathrm{d} R_{i}}{\mathrm{~d} V}=\frac{1}{V} \frac{\mathrm{~d} n_{i}}{\mathrm{~d} \tau} \tag{9}
\end{align*}
$$

CSTR balance for reactor 1 , component A ( $F$ is volume flow rate, assuming constant $\rho$ ):

$$
\begin{equation*}
F C_{\mathrm{A} 0}-F C_{\mathrm{A} 1}+\nu_{\mathrm{A}} r\left(C_{\mathrm{A} 1}\right) V=0 \tag{10}
\end{equation*}
$$

Power law with equilibrium for reaction $\mathrm{A}+\mathrm{B} \rightleftharpoons \mathrm{C}+\mathrm{D}$, where R is backwards reaction:

$$
\begin{equation*}
r=k c_{\mathrm{A}}^{a} c_{\mathrm{B}}^{b}-k_{\mathrm{R}} c_{\mathrm{C}}^{c} c_{\mathrm{D}}^{d} \tag{11}
\end{equation*}
$$

Langmuir-Hinschelwood for surface reaction $A+B \rightleftharpoons C$ :

$$
\begin{equation*}
r=k_{1} q_{\mathrm{A}} q_{\mathrm{B}}-k_{2} q_{\mathrm{C}}=k_{1} K_{\mathrm{A}} c_{\mathrm{A}} K_{\mathrm{B}} c_{\mathrm{B}} q^{2}-k_{2} K_{\mathrm{c}} c_{\mathrm{c}} q \tag{12}
\end{equation*}
$$

Where $q$ (surface conc. of free active sites $\left[\frac{\mathrm{mol}}{\mathrm{m}^{2}}\right]$ ) is following ( $Q$ is total surface conc.):

$$
\begin{equation*}
q=\frac{Q}{1+K_{\mathrm{A}} c_{\mathrm{A}}+K_{\mathrm{B}} c_{\mathrm{B}}+K_{\mathrm{C}} c_{\mathrm{C}}} \tag{13}
\end{equation*}
$$

Derivation: $\left(q, q_{\mathrm{A}}, q_{\mathrm{B}}, q_{\mathrm{C}}\right.$ are unknowns, $Q, K_{\mathrm{A}}, K_{\mathrm{B}}, K_{\mathrm{C}}$ are parameters)

$$
\begin{align*}
Q & =q+q_{\mathrm{A}}+q_{\mathrm{B}}+q_{\mathrm{C}}  \tag{14}\\
K_{i} & =\frac{q_{i}}{c_{i} q} \tag{15}
\end{align*}
$$

Michaelis-Menten for enzymatic reaction $\mathrm{E}+\mathrm{S} \rightleftharpoons \mathrm{ES} \rightarrow \mathrm{P}:$

$$
\begin{equation*}
r_{\mathrm{P}}=k_{\mathrm{MAX}} \frac{c_{\mathrm{S}}}{k_{\mathrm{A}}+c_{\mathrm{S}}} \tag{16}
\end{equation*}
$$

Derivation: $\mathrm{d} c_{\mathrm{ES}} / \mathrm{d} \tau=0, \quad c_{\mathrm{E}}=c_{\mathrm{E}}^{0}-c_{\mathrm{ES}}$

## Thermodynamics

Antoine equation for vapor pressure of pure component $i$ (tlak sytých par):

$$
\begin{equation*}
\ln p_{i}^{\circ}=A-\frac{B}{T+C} \tag{17}
\end{equation*}
$$

Dependence of various constants on temperature:

$$
\begin{equation*}
\frac{\mathrm{d} \ln k}{\mathrm{~d} T}=\frac{\Delta H}{R T^{2}} \tag{18}
\end{equation*}
$$

| constant | equilibrium | reaction rate | $p^{\circ}$ | Henry | adsorption |
| :--- | :---: | :---: | :---: | :---: | :---: |
| enthalpy | reaction | activation energy | evaporation | dissolution | adsorption |

## Dimensionless numbers

Reynolds = ratio of momentum (hybnost) / viscous forces (vazké síly).
Eq. 19 for pipe (or a particle, then $d=d_{p}$ ), 20 for rotary component:

$$
\begin{align*}
\operatorname{Re} & =\frac{\rho v d}{\eta}  \tag{19}\\
\operatorname{Re} & =\frac{\rho f d^{2}}{\eta} \tag{20}
\end{align*}
$$

Note: $\operatorname{Re}<2300$ laminar, $\operatorname{Re}>10000$ turbulent.
Schmidt $=$ ratio of convective diffusion / molecular diffusion:

$$
\begin{equation*}
\mathrm{Sc}=\frac{\nu}{D}=\frac{\eta}{\rho D} \tag{21}
\end{equation*}
$$

Prandtl = ratio of viscosity $/$ thermal diffusivity (teplotní difuzivita), where $\lambda$ is thermal conductivity (tepelná vodivost) $\left[\frac{\mathrm{W}}{\mathrm{m} \cdot \mathrm{K}}\right]$ :

$$
\begin{equation*}
\operatorname{Pr}=\frac{\nu}{D_{\mathrm{T}}}=\frac{\eta / \rho}{\lambda / C_{\mathrm{p}} / \rho}=\frac{\eta C_{\mathrm{p}}}{\lambda} \tag{22}
\end{equation*}
$$

Grasshof describes the combined effect of gravity + thermal expansion (the free convection):

$$
\begin{equation*}
\mathrm{Gr}=\frac{g L^{3}}{\nu^{2}} \beta \Delta T \quad \beta=\frac{1}{V_{m}} \frac{\partial V_{m}}{\partial T} \tag{23}
\end{equation*}
$$

Nusselt $=$ ratio of convection / conduction (vedení), $\alpha$ is heat transfer coeff:

$$
\begin{equation*}
\mathrm{Nu}=\frac{\alpha d}{\lambda} \tag{24}
\end{equation*}
$$

Usually $\mathrm{Nu}=\mathrm{Nu}(\operatorname{Re}, \mathrm{Pr}, \mathrm{Gr})$; for a smooth tubular pipe you may use the Dittus-Boelter correlation: $\mathrm{Nu}=0.023 \mathrm{Re}^{0.8} \mathrm{Pr}^{n}$, where $n$ is 0.3 when cooling, 0.4 when heating.

Sherwood $=$ ratio of convection $/$ diffusion, $\beta$ is mass transfer coeff (koef. přestupu hmoty):

$$
\begin{equation*}
\mathrm{Sh}=\frac{\beta d}{D} \tag{25}
\end{equation*}
$$

Power number for a rotary component:

$$
\begin{equation*}
\mathrm{N}_{\mathrm{P}}=\frac{P}{\rho d^{5} f^{3}} \tag{26}
\end{equation*}
$$

Péclet relates to axial dispersion coeff $E$ :

$$
\begin{equation*}
\mathrm{Pe}=\frac{l v}{E} \tag{27}
\end{equation*}
$$

Damköhler $=$ ratio of component A formation rate / convective transport rate in a tubular reactor, generally and for first order reaction:

$$
\begin{equation*}
\mathrm{Da}=\frac{L r}{v c_{\mathrm{A}}} \quad=\frac{L k}{v} \tag{28}
\end{equation*}
$$

## Hydraulics

Bernoulli equation in various dimensions for incompressible flow.
Equation 29 serves no practical purpose, but is a great mnemotechnic.
Divide it with either $m, V, m g$ to obtain equations 30, 31, 32.

$$
\begin{align*}
\frac{1}{2} m v_{1}^{2}+m g h_{1}+p_{1} V & =\frac{1}{2} m v_{2}^{2}+m g h_{2}+p_{2} V & & {[\mathrm{~J}] }  \tag{29}\\
\frac{1}{2} v_{1}^{2}+g h_{1}+p_{1} / \rho & =\frac{1}{2} v_{2}^{2}+g h_{2}+p_{2} / \rho & & {[\mathrm{J} / \mathrm{kg}] }  \tag{30}\\
\frac{1}{2} \rho v_{1}^{2}+\rho g h_{1}+p_{1} & =\frac{1}{2} \rho v_{2}^{2}+\rho g h_{2}+p_{2} & & {[\mathrm{~Pa}] }  \tag{31}\\
\frac{1}{2} \frac{v_{1}^{2}}{g}+h_{1}+\frac{p_{1}}{\rho g} & =\frac{1}{2} \frac{v_{2}^{2}}{g}+h_{2}+\frac{p_{2}}{\rho g} & & {[\mathrm{~m}] } \tag{32}
\end{align*}
$$

Energy dissipation can be added to the left-hand side of 30 for either a straight pipe section of length $L$ and diameter $d$, or generally any pipe fitting (tvarovka či armatura):

$$
\begin{equation*}
e_{\mathrm{dis}}=\lambda \frac{L v^{2}}{2 d} \quad=\zeta \frac{v^{2}}{2} \quad[\mathrm{~J} / \mathrm{kg}] \tag{33}
\end{equation*}
$$

Note 1: friction coeff $\lambda$ and local resistance coeff $\zeta$ are dimensionless
Note 2: for laminar flow and straight tubular pipe, $\lambda=64 / R e$
By combining eqs 30,33 and $v=F / A$, we get pressure drop as function of flow rate $F$ :

$$
\begin{equation*}
\Delta p=e_{\mathrm{dis}} \rho=\lambda \rho \frac{8 L F^{2}}{\pi^{2} d^{5}} \tag{34}
\end{equation*}
$$

## Valve sizing:

$C_{V}$ means such flow rate $F[\mathrm{gal} / \mathrm{min}]$ of water at $60^{\circ} \mathrm{F}$, that $\Delta P=1 \mathrm{psi}$
$K_{V}$ means such flow rate $F\left[\mathrm{~m}^{3} / \mathrm{h}\right]$ of water at $16^{\circ} \mathrm{C}$, that $\Delta P=1$ bar
$K_{V} \doteq 0.8650 C_{V}$
By combining eqs 30, 33, and $\rho_{\mathrm{r}}=\rho / \rho_{\mathrm{H}_{2} \mathrm{O}}$ we get the pressure drop in bars:

$$
\begin{equation*}
\frac{\Delta p}{\mathrm{bar}}=\rho_{\mathrm{r}} \frac{F^{2}}{K_{V}^{2}} \tag{35}
\end{equation*}
$$

Note: make sure to use the same units for $K_{V}$ and $F$.
Ergun equation for $\Delta p$ along axis $x$ in packed columns (výplňové kolony), where $\varepsilon$ is void fraction (mezerovitost), $d_{\mathrm{e}}$ is equivalent particle diameter, $v_{x}$ is superficial velocity. For compressible flow: $\mu=\mu(x), \rho=\rho(x)$, but for incompressible we may simplify: $\frac{\partial p}{\partial x}=\frac{\Delta p}{L}$

$$
\begin{equation*}
\frac{\partial p}{\partial x}=\frac{150 \mu}{d_{\mathrm{e}}^{2}} \frac{(1-\varepsilon)^{2}}{\varepsilon^{3}} v_{x}+\frac{7}{4} \frac{\rho}{d_{\mathrm{e}}} \frac{(1-\varepsilon)}{\varepsilon^{3}} v_{x}\left|v_{x}\right| \tag{36}
\end{equation*}
$$

Particle settling (usazování částic) in laminar flow (calculate settling velocity from Re):

$$
\begin{equation*}
\operatorname{Re}=\frac{\operatorname{Ar}^{2}}{18^{3}} \quad \operatorname{Ar}=\frac{d_{p}^{3} g}{\nu^{2}} \frac{\left(\rho_{p}-\rho\right)}{\rho} \tag{37}
\end{equation*}
$$

Note: verify that $\operatorname{Re}<2300$ (otherwise use more complex calculation)

## Mass \& heat effects

Adiabatic reaction exotherm (nárůst teploty), where $\hat{C}_{p, \mathrm{sp}}\left[\frac{\mathrm{J}}{\mathrm{kg} \cdot \mathrm{K}}\right]$ :

$$
\begin{equation*}
\Delta T_{\mathrm{ad}}=\frac{-\Delta H_{\mathrm{R}}}{\hat{C}_{p, \mathrm{sp}}} \frac{c_{\mathrm{A}}^{0}}{\rho} X \tag{38}
\end{equation*}
$$

Plate exchanger (deskový výměník) heat transfer from fluid 1 to fluid 2 through solid wall of surface $A$, where $K, \alpha$ are heat transfer coeffs $\left[\frac{\mathrm{W}}{\mathrm{m}^{2} \mathrm{~K}}\right]$ :

$$
\begin{align*}
\dot{Q} & =K A \Delta T  \tag{39}\\
\frac{1}{K} & =\frac{1}{\alpha_{1}}+\frac{d}{\lambda}+\frac{1}{\alpha_{2}} \tag{40}
\end{align*}
$$

Tube exchanger (trubkový výměník) with inner tube diameter $d_{1}$, outer $d_{2}$, where $K_{L}$ is heat transfer coeff per length $\left[\frac{\mathrm{W}}{\mathrm{m} \cdot \mathrm{K}}\right]$ :

$$
\begin{align*}
\dot{Q} & =K_{L} L \Delta T  \tag{41}\\
\frac{2 \pi}{K_{L}} & =\frac{1}{d_{1} \alpha_{1}}+\frac{1}{\lambda} \ln \frac{d_{2}}{d_{1}}+\frac{1}{d_{2} \alpha_{2}} \tag{42}
\end{align*}
$$

$\Delta T$ in countercurrent exchange between points $\mathrm{A}, \mathrm{B}$, where $\Delta T_{A}$ is the temp. difference between the two mediums at point A, and analog. for B:

$$
\begin{equation*}
\Delta T_{\mathrm{ls}}=\frac{\Delta T_{\mathrm{A}}-\Delta T_{\mathrm{B}}}{\ln \frac{\Delta T_{\mathrm{A}}}{\Delta T_{\mathrm{B}}}} \tag{43}
\end{equation*}
$$

Mass transfer of component $i$ from fluid phase $x$ to fluid phase $y$ through phase interface, expressed for $c_{i}\left[\mathrm{~mol} / \mathrm{m}^{3}\right]$, where $K$ are mass transfer coeffs $[\mathrm{m} / \mathrm{s}]$,
$m$ is distribution coeff at interface: $m=c_{y, \text { eq }} / c_{x, \text { eq }}$

$$
\begin{align*}
\dot{n}_{i} & =K_{x} A\left(c_{x}-c_{y} / m\right) & \dot{n}_{i}=K_{y} A\left(c_{x} m-c_{y}\right)  \tag{44}\\
\frac{1}{K_{x}} & =\frac{1}{\beta_{x}}+\frac{1}{\beta_{y} m} & \frac{1}{K_{y}}=\frac{1}{\beta_{x} / m}+\frac{1}{\beta_{y}} \tag{45}
\end{align*}
$$

Note: choose only one from eqs 44, as they are lin. dependent; $K_{x}=m K_{y}$

## Porous solids and catalysis

Throughput (zatížení) of a fixed-bed catalytic reactor is often expressed via LHSV [ $\mathrm{h}^{-1}$ ] (liquid hourly space velocity) for liquid feed incl. trickle-bed (skrápěné lože),
and for gas feed via $G H S V$, where $\bar{M}_{\text {feed }}$ is mean molar mass, $p^{\circ}=\mathrm{atm}, T^{\circ}=273.15 \mathrm{~K}$

$$
\begin{align*}
L H S V & =\frac{\dot{\text { feed }}^{V_{\text {cat }}}}{=} \frac{\dot{m}_{\text {feed }}}{\rho_{\text {feed }} V_{\text {cat }}}  \tag{46}\\
G H S V & =\frac{\dot{V}_{\text {feed }}^{\circ}}{V_{\text {cat }}}=\frac{\dot{n}_{\text {feed }}}{V_{\text {cat }}} \frac{R T^{\circ}}{p^{\circ}}=\frac{\dot{m}_{\text {feed }}}{V_{\text {cat }}} \frac{R T^{\circ}}{\bar{M}_{\text {feed }} p^{\circ}} \tag{47}
\end{align*}
$$

Thiele modulus (dimensionless) $=$ ratio of surface reaction rate $/$ mass transfer, $c_{\mathrm{A}, \mathrm{S}}$ is $c$ at catalyst surface, $d_{p}$ is radius of particle. The second form is simplification when $r=k S c_{\mathrm{A}, \mathrm{S}}$, with $S$ being the specific surface $\left[\mathrm{m}^{-1}\right]$.

$$
\begin{equation*}
\kappa=d_{p} \sqrt{\frac{r}{c_{\mathrm{A}, \mathrm{~S}} D_{\mathrm{eff}}}}=d_{p} \sqrt{\frac{k S}{D_{\mathrm{eff}}}} \tag{48}
\end{equation*}
$$

Viscous flow in a porous particle or membrane, where $\beta$ is the permeability $\left[\mathrm{m}^{2}\right]$

$$
\begin{equation*}
\dot{V}=\beta \frac{A}{L} \frac{\Delta p}{\eta} \tag{49}
\end{equation*}
$$

Freundlich isotherm (adsorption of component $i$ ) with parameters $K, n$ :

$$
\begin{equation*}
\frac{m_{i}}{m_{\text {adsorbent }}}=K p^{\frac{1}{n}} \tag{50}
\end{equation*}
$$

Langmuir-Hinschelwood: see eq $\mathbf{1 2}$ (which integrates the isotherm in the reaction rate formula)

