Cheming cheatsheet

A handy guide containing the most important equations that a chemical engineer will need in practical life. For a few selected equations, their derivation process is outlined, but only in *very* brief points. Czech names are in violet.

Unit conversions are not included; see <u>this tool</u> that will tackle anything you come across. By Jiří Zbytovský in 2023

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Reaction quantification

E, P stands for an extensive quantity for educts (reaktanty) and products, such as n_i , m_i , \dot{n}_i . When certain conditions are met, also intensive quantities may be balanced: c_i , p_i , w_i etc... ν_i is the stoichiometric coefficient of a component *i*.

Extent of reaction (reakční rozsah) for a component i

$$\xi = \frac{n_i - n_i^0}{\nu_i} \qquad \qquad \xi_V = \frac{n_i - n_i^0}{V\nu_i} \tag{1}$$

Conversion

$$X = \frac{E^0 - E}{E^0} \tag{2}$$

Yield (výtěžek)

$$Y = \frac{P}{E^0} \frac{\nu_{\rm E}}{\nu_{\rm P}} = X \cdot S \tag{3}$$

Selectivity (of one reaction with st. coeffs $\nu_{\rm E}$, $\nu_{\rm P}$ against all other reactions)

$$S = \frac{P}{E^0 - E} \frac{\nu_{\rm E}}{\nu_{\rm P}} \tag{4}$$

Relative selectivity (of two parallel reactions $E \to P, P*$ with st. coeffs ν_P, ν_{P*})

$$S_{\rm P/P*} = \frac{P}{P*} \frac{\nu_{\rm P}*}{\nu_{\rm P}} = \frac{r_{\rm P}}{r_{\rm P*}}$$
(5)

Reaction kinetics

Various definitions of **reaction rate** (reakční rychlost):

$$R = \frac{\mathrm{d}\xi}{\mathrm{d}\tau} \tag{6}$$

$$R_i = R\nu_i \tag{7}$$

$$r = \frac{\mathrm{d}R}{\mathrm{d}V} = \frac{\mathrm{d}\xi_V}{\mathrm{d}\tau} = \frac{1}{V\nu_i}\frac{\mathrm{d}n_i}{\mathrm{d}\tau} \tag{8}$$

$$r_i = r\nu_i = \frac{\mathrm{d}R_i}{\mathrm{d}V} = \frac{1}{V}\frac{\mathrm{d}n_i}{\mathrm{d}\tau} \tag{9}$$

CSTR balance for reactor 1, component A (*F* is volume flow rate, assuming constant ρ):

$$FC_{A0} - FC_{A1} + \nu_A r(C_{A1})V = 0$$
(10)

Power law with equilibrium for reaction $A + B \rightleftharpoons C + D$, where R is backwards reaction:

$$r = kc_{\rm A}^a c_{\rm B}^b - k_{\rm R} c_{\rm C}^c c_{\rm D}^d \tag{11}$$

Langmuir-Hinschelwood for surface reaction $A + B \rightleftharpoons C$:

$$r = k_1 q_{\rm A} q_{\rm B} - k_2 q_{\rm C} = k_1 K_{\rm A} c_{\rm A} K_{\rm B} c_{\rm B} q^2 - k_2 K_{\rm c} c_{\rm c} q$$
(12)

Where q (surface conc. of free active sites $\left[\frac{\text{mol}}{\text{m}^2}\right]$) is following (Q is total surface conc.):

$$q = \frac{Q}{1 + K_{\rm A}c_{\rm A} + K_{\rm B}c_{\rm B} + K_{\rm C}c_{\rm C}} \tag{13}$$

Derivation: $(q, q_A, q_B, q_C \text{ are unknowns}, Q, K_A, K_B, K_C \text{ are parameters})$

$$Q = q + q_{\rm A} + q_{\rm B} + q_{\rm C} \tag{14}$$

$$K_i = \frac{q_i}{c_i q} \tag{15}$$

Michaelis-Menten for enzymatic reaction $E + S \rightleftharpoons ES \rightarrow P$:

$$r_{\rm P} = k_{\rm MAX} \frac{c_{\rm S}}{k_{\rm A} + c_{\rm S}} \tag{16}$$

Derivation: $dc_{\rm ES}/d\tau = 0$, $c_{\rm E} = c_{\rm E}^0 - c_{\rm ES}$

Thermodynamics

Antoine equation for vapor pressure of pure component i (tlak sytých par):

$$\ln p_i^\circ = A - \frac{B}{T+C} \tag{17}$$

Dependence of various constants on temperature:

$$\frac{\mathrm{d}\ln k}{\mathrm{d}T} = \frac{\Delta H}{RT^2} \tag{18}$$

constant	equilibrium	reaction rate	p°	Henry	adsorption
enthalpy	reaction	activation energy	evaporation	dissolution	adsorption

Dimensionless numbers

Reynolds = ratio of momentum (hybnost) / viscous forces (vazké síly). Eq. 19 for pipe (or a particle, then $d = d_p$), 20 for rotary component:

$$Re = \frac{\rho v d}{\eta} \tag{19}$$

$$\operatorname{Re} = \frac{\rho f d^2}{\eta} \tag{20}$$

Note: Re < 2300 laminar, Re > 10000 turbulent.

Schmidt = ratio of convective diffusion / molecular diffusion:

$$Sc = \frac{\nu}{D} = \frac{\eta}{\rho D} \tag{21}$$

Prandtl = ratio of viscosity / thermal diffusivity (teplotní difuzivita), where λ is thermal conductivity (tepelná vodivost) $\left[\frac{W}{m \cdot K}\right]$:

$$\Pr = \frac{\nu}{D_{\rm T}} = \frac{\eta/\rho}{\lambda/C_{\rm p}/\rho} = \frac{\eta C_{\rm p}}{\lambda}$$
(22)

Grasshof describes the combined effect of gravity + thermal expansion (the free convection):

$$Gr = \frac{gL^3}{\nu^2} \beta \Delta T \qquad \qquad \beta = \frac{1}{V_m} \frac{\partial V_m}{\partial T}$$
(23)

Nusselt = ratio of convection / conduction (vedení), α is heat transfer coeff:

$$Nu = \frac{\alpha d}{\lambda} \tag{24}$$

Usually Nu = Nu(Re, Pr, Gr); for a smooth tubular pipe you may use the Dittus-Boelter correlation: Nu = $0.023 \text{Re}^{0.8} \text{Pr}^n$, where n is 0.3 when cooling, 0.4 when heating.

Sherwood = ratio of convection / diffusion, β is mass transfer coeff (koef. přestupu hmoty):

$$Sh = \frac{\beta d}{D} \tag{25}$$

Power number for a rotary component:

$$N_{\rm P} = \frac{P}{\rho d^5 f^3} \tag{26}$$

Péclet relates to axial dispersion coeff *E*:

$$Pe = \frac{lv}{E} \tag{27}$$

Damköhler = ratio of component A formation rate / convective transport rate in a tubular reactor, generally and for first order reaction:

$$Da = \frac{Lr}{vc_A} \qquad \qquad = \frac{Lk}{v} \tag{28}$$

Hydraulics

Bernoulli equation in various dimensions for *incompressible* flow.

Equation 29 serves no practical purpose, but is a great mnemotechnic.

Divide it with either m, V, mg to obtain equations 30, 31, 32.

$$\frac{1}{2}mv_1^2 + mgh_1 + p_1V = \frac{1}{2}mv_2^2 + mgh_2 + p_2V \qquad [J]$$

$$\frac{1}{2}v_1^2 + gh_1 + p_1/\rho = \frac{1}{2}v_2^2 + gh_2 + p_2/\rho \qquad [J/kg] \qquad (30)$$

$$\frac{1}{2}\rho v_1^2 + \rho g h_1 + p_1 = \frac{1}{2}\rho v_2^2 + \rho g h_2 + p_2$$
 [Pa] (31)

$$\frac{1}{2}\frac{v_1^2}{g} + h_1 + \frac{p_1}{\rho g} = \frac{1}{2}\frac{v_2^2}{g} + h_2 + \frac{p_2}{\rho g}$$
 [m] (32)

Energy dissipation can be added to the left-hand side of 30 for either a straight pipe section of length L and diameter d, or generally any pipe fitting (tvarovka či armatura):

Note 1: friction coeff λ and local resistance coeff ζ are dimensionless Note 2: for laminar flow and straight tubular pipe, $\lambda = 64/Re$

By combining eqs 30, 33 and v = F/A, we get pressure drop as function of flow rate F:

$$\Delta p = e_{\rm dis}\rho = \lambda \rho \frac{8LF^2}{\pi^2 d^5} \tag{34}$$

Valve sizing:

 C_V means such flow rate F [gal/min] of water at 60°F, that $\Delta P = 1$ psi K_V means such flow rate F [m³/h] of water at 16°C, that $\Delta P = 1$ bar $K_V \doteq 0.8650 C_V$

By combining eqs 30, 33, and $\rho_r = \rho/\rho_{H_2O}$ we get the pressure drop in bars:

$$\frac{\Delta p}{\text{bar}} = \rho_{\rm r} \frac{F^2}{K_V^2} \tag{35}$$

Note: make sure to use the same units for K_V and F.

Ergun equation for Δp along axis x in packed columns (výplňové kolony), where ε is void fraction (mezerovitost), d_e is equivalent particle diameter, v_x is superficial velocity. For *compressible* flow: $\mu = \mu(x), \rho = \rho(x)$, but for *incompressible* we may simplify: $\frac{\partial p}{\partial x} = \frac{\Delta p}{L}$

$$\frac{\partial p}{\partial x} = \frac{150\mu}{d_{\rm e}^2} \frac{(1-\varepsilon)^2}{\varepsilon^3} v_x + \frac{7}{4} \frac{\rho}{d_{\rm e}} \frac{(1-\varepsilon)}{\varepsilon^3} v_x |v_x| \tag{36}$$

Particle settling (usazování částic) in laminar flow (calculate settling velocity from Re):

$$\operatorname{Re} = \frac{\operatorname{Ar}^2}{18^3} \qquad \qquad \operatorname{Ar} = \frac{d_p^3 g}{\nu^2} \frac{(\rho_p - \rho)}{\rho} \qquad (37)$$

Note: verify that Re < 2300 (otherwise use more complex calculation)

Mass & heat effects

Adiabatic reaction exotherm (nárůst teploty), where $\hat{C}_{p,sp}$ [$\frac{\mathbf{J}}{\mathrm{kg}\cdot\mathbf{K}}$]:

$$\Delta T_{\rm ad} = \frac{-\Delta H_{\rm R}}{\hat{C}_{p,\rm sp}} \frac{c_{\rm A}^0}{\rho} X \tag{38}$$

Plate exchanger (deskový výměník) heat transfer from fluid 1 to fluid 2 through solid wall of surface A, where K, α are heat transfer coeffs $\left[\frac{W}{m^2 K}\right]$:

$$\dot{Q} = KA\Delta T \tag{39}$$

$$\frac{1}{K} = \frac{1}{\alpha_1} + \frac{d}{\lambda} + \frac{1}{\alpha_2} \tag{40}$$

Tube exchanger (trubkový výměník) with inner tube diameter d_1 , outer d_2 , where K_L is heat transfer coeff per length $\left[\frac{W}{m \cdot K}\right]$:

$$\dot{Q} = K_L L \Delta T \tag{41}$$

$$\frac{2\pi}{K_L} = \frac{1}{d_1\alpha_1} + \frac{1}{\lambda}\ln\frac{d_2}{d_1} + \frac{1}{d_2\alpha_2}$$
(42)

 ΔT in countercurrent exchange between points A, B, where ΔT_A is the temp. difference between the two mediums at point A, and analog. for B:

$$\Delta T_{\rm ls} = \frac{\Delta T_{\rm A} - \Delta T_{\rm B}}{\ln \frac{\Delta T_{\rm A}}{\Delta T_{\rm B}}} \tag{43}$$

Mass transfer of component *i* from fluid phase *x* to fluid phase *y* through phase interface, expressed for $c_i [\text{mol/m}^3]$, where *K* are mass transfer coeffs [m/s], *m* is distribution coeff *at interface*: $m = c_{y,\text{eq}}/c_{x,\text{eq}}$

$$\dot{n}_{i} = K_{x}A\left(c_{x} - c_{y}/m\right) \qquad \dot{n}_{i} = K_{y}A\left(c_{x}m - c_{y}\right) \tag{44}$$

$$\frac{1}{K_x} = \frac{1}{\beta_x} + \frac{1}{\beta_y m} \qquad \qquad \frac{1}{K_y} = \frac{1}{\beta_x/m} + \frac{1}{\beta_y}$$
(45)

Note: choose only one from eqs 44, as they are lin. dependent; $K_x = mK_y$

Porous solids and catalysis

Throughput (zatížení) of a fixed-bed catalytic reactor is often expressed via LHSV [h⁻¹] (liquid hourly space velocity) for liquid feed *incl. trickle-bed (skrápěné lože)*, and for gas feed via GHSV, where $\overline{M}_{\text{feed}}$ is mean molar mass, $p^{\circ} = \text{atm}, T^{\circ} = 273.15\text{K}$

$$LHSV = \frac{\dot{V}_{\text{feed}}}{V_{\text{cat}}} = \frac{\dot{m}_{\text{feed}}}{\rho_{\text{feed}}V_{\text{cat}}}$$
(46)

$$GHSV = \frac{\dot{V}_{\text{feed}}^{\circ}}{V_{\text{cat}}} = \frac{\dot{n}_{\text{feed}}}{V_{\text{cat}}} \frac{RT^{\circ}}{p^{\circ}} = \frac{\dot{m}_{\text{feed}}}{V_{\text{cat}}} \frac{RT^{\circ}}{\bar{M}_{\text{feed}}p^{\circ}}$$
(47)

Thiele modulus (dimensionless) = ratio of surface reaction rate / mass transfer, $c_{A,S}$ is c at catalyst surface, d_p is radius of particle. The second form is simplification when $r = kSc_{A,S}$, with S being the specific surface $[m^{-1}]$.

$$\kappa = d_p \sqrt{\frac{r}{c_{\rm A,S} D_{\rm eff}}} = d_p \sqrt{\frac{kS}{D_{\rm eff}}}$$
(48)

Viscous flow in a porous particle or membrane, where β is the permeability $[m^2]$

$$\dot{V} = \beta \frac{A}{L} \frac{\Delta p}{\eta} \tag{49}$$

Freundlich isotherm (adsorption of component i) with parameters K, n:

$$\frac{m_i}{m_{\rm adsorbent}} = K p^{\frac{1}{n}} \tag{50}$$

Langmuir-Hinschelwood: see eq 12 (which integrates the isotherm in the reaction rate formula)